E & H → Electric & magnetic field intensities
V/m amp/m.

D & B → Electric & magnetic flux densities
coloumb/m^2 Weber/m^2
tesla.

D → Electric displacement

B → Magnetic induction.

ρ & J → volume charge density
& Electric current density
(charge flux) of any external charges (i.e. not including any polarization charges & currents)

Maxwell's equations.

There are four fundamental equations of electromagnetism known as Maxwell's equations which may be written in differential form as —

① $\nabla \cdot D = \rho$ (Differential form of Gauss's law in electrostatics)

② $\nabla \cdot B = 0$ (Differential form of Gauss's law in magnetostatics)

③ $\nabla \times E = -\dfrac{\partial B}{\partial t}$ (Differential form of Faraday's law of electromagnetic induction)

④ $\nabla \times H = J + \dfrac{\partial D}{\partial t}$ (Maxwell's modification of Ampere's law)

D → Electric displacement vector in coloumb/m^2

ρ → charge density coul/m^3

B → Magnetic induction weber/m^2

E → Electric field intensity volt/m

H → Magnetic field intensity in amp/m-turn

Using Gauss's divergence theorem to change surface integral into volume integral

$$\iiint_V \operatorname{div}(\epsilon_0 E)\cdot dv = \iiint_V \rho \cdot dv - \iiint_V \operatorname{div} P \cdot dv$$

$$\iiint_V \operatorname{div}(\epsilon_0 E + P)\cdot dv = \iiint_V \rho \cdot dv$$

But $\epsilon_0 E + P = D$

where D is electric displacement vector.

Thus $$\iiint_V \operatorname{div} D \cdot dv = \iiint_V \rho \cdot dv.$$

$$\iiint_V (\operatorname{div} D - \rho)\cdot dv = 0$$

Since the equation is true for any volume, integrant in this equation must vanish

$\therefore \ \mathrm{div}\, D - \rho = 0$

$\therefore \ \mathrm{div}\, D = \rho$

i.e. $\nabla \cdot D = \rho$

which is Maxwell's first equation.

(2) Derivation of Maxwell's second equation

$$\mathrm{div}\, B = \nabla \cdot B = 0.$$

Since isolated magnetic poles & magnetic current due to them have no physical significance. Therefore magnetic lines of force in general are ~~other~~ closed loops. Consequently the number of magnetic lines of force entering any arbitrary closed surface is exactly same as lines of force leaving that area. It means that the flux of magnetic induction B across any closed surface is always zero i.e.

$$\int_s B \cdot ds = 0$$

(1) Derivation of first equation

$$\text{Div } D = \rho \quad \nabla \cdot D = \rho$$

Let us consider surface S bounding volume V in dielectric medium. In a dielectric medium total charge is sum of free charges ~~plus~~ & polarization charges. If ρ & ρ_P are charge densities of free charge & polarization charge at a point in a small volume dv, then Gauss's law can be expressed as —

$$\iint_S E \cdot ds = \frac{1}{\epsilon_0} \iiint_V (\rho + \rho_P) \cdot dv$$

But polarization charge density $\rho_P = -\text{div } P$,

thus

$$\iint_S E \cdot ds = \frac{1}{\epsilon_0} \iiint_V (\rho - \text{div } P) \cdot dv$$

$$\iint_S \epsilon_0 E \cdot ds = \iiint_V \rho \cdot dv - \iiint_V \text{div } P \cdot dv$$

(3) Derivation of Maxwell's third eqn

(36)

$$\text{curl } E = -\frac{\partial B}{\partial t}$$

According to Faraday's law of electromagnetic induction it is known that e.m.f. induced in closed loop is defined as negative rate of ~~..~~ change of magnetic flux

$$\varepsilon = -\frac{d\phi}{dt}$$

But magnetic flux $\phi = \iint_S B \cdot ds$

where S is closed surface.

$$\therefore \quad \varepsilon = -\frac{d}{dt}\iint_S B \cdot ds$$

$$\varepsilon = -\iint_S \frac{\partial B}{\partial t} \cdot ds \longrightarrow (1)$$

Since in fix space, B changes only w.r.t. time

But emf can also be computed by calculating the work done in carrying a unit charge round the close loop c. Thus if E is electric field intensity at a small element dl of loop, we have

$$e = \oint_C E \cdot dl \longrightarrow (2)$$

comparing eqn (1) & (2)

$$\oint_C E \cdot dl = -\iint_S \frac{\partial B}{\partial t} \cdot ds$$

Using Stokes theorem to change line integral into surface integral, we get

$$\iint_S \text{curl}\, E \cdot ds = -\iint_S \frac{\partial B}{\partial t} \cdot ds$$

$$\iint \left(\text{curl}\, E + \frac{\partial B}{\partial t} \right) \cdot ds = 0$$

using Gauss's divergence theorem we can convert surface integral to volume integral as —

$$\iiint_V \text{div } B \cdot dv = 0$$

As surface S bounding this volume is arbitrary the integrant must vanish

$$\therefore \text{div } B = 0$$

$$\therefore \nabla \cdot B = 0$$

This is Maxwell's second eqn

left side is loosing same charge due to current flowing into surface S then same amount of charge must be gained by right side.

In magnetostatics, the currents are steady currents — $\frac{d\rho}{dt} = 0$ no accumulation of charge

$\bar{\nabla} \cdot J = 0$ · Steady state form (Relativistic valid)

Coloumb's law

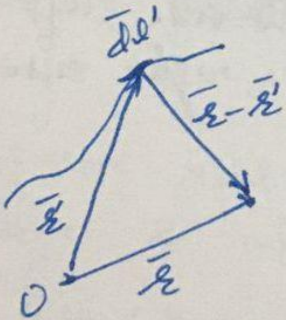
$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Biot - Savart's law.

source of B is current distribution.

Field at dist. $\bar{r}$ due to current element $\overrightarrow{dl}$ carrying current I

$$d\bar{B}(r) = I \frac{\mu_0}{4\pi} \frac{\overrightarrow{dl} \times (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3}$$



$\mu_0 \rightarrow$ Permeability of free space

$$\bar{B}(r) = \frac{\mu_0}{4\pi} \int \frac{\bar{J}(\bar{r}') \times (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3} d^3 r'$$

current density I time dl is equal to $\bar{J}$ times ds.

$$= \frac{\mu_0}{4\pi} \int \left[\nabla \left(\frac{1}{|\bar{r} - \bar{r}'|} \right) \times \bar{J}(\bar{r}') \right] d^3 \bar{r}'$$

$$= \frac{\mu_0}{4\pi} \int \left[\nabla \times \left(\frac{1}{|\bar{r} - \bar{r}'|} \bar{J}(\bar{r}') \right) \right] d^3 \bar{r}'$$

$$\bar{B}(r) = \frac{\mu_0}{4\pi} \nabla \times \int \frac{\bar{J}(\bar{r}')}{|\bar{r} - \bar{r}'|} d^3r'$$

Divergence of curl is zero

$$\therefore \nabla B = \mathrm{div}\left(\frac{\mu_0}{4\pi} \nabla \times \int \right)$$

curl of some quantity.

$\nabla \cdot B = 0$ — Absence of monopoles.

In electrostatic $\nabla \cdot E = \frac{\rho}{\epsilon_0}$.

A vector field is uniquely specified by divergence & curl.

$$\mathrm{curl}\, B = \frac{\mu_0}{4\pi} \nabla \times$$

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{A}) = \nabla(\nabla \cdot A) - \nabla^2 A$$

$$\nabla \times B = \frac{\mu_0}{4\pi} \nabla \times \left(\nabla \times \int \frac{\bar{J}(\bar{r}')}{|\bar{r} - \bar{r}'|} d^3r' \right)$$

$$= \frac{\mu_0}{4\pi} \left[\nabla \left(\nabla \cdot \int \frac{\bar{J}(\bar{r}')}{|r - r'|} \cdot d^3r' \right) - \bar{\nabla}^2 \int \frac{\bar{J}(r')}{|\bar{r} - \bar{r}'|} d^3r' \right]$$

Since the surface is arbitrary the eqn holds only if integrant vanish, (37)

$$\text{curl}\, E + \frac{\partial B}{\partial t} = 0$$

$$\therefore \text{curl}\, E = -\frac{\partial B}{\partial t}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

This is Maxwell's third eqn.

Electrostatic - field by static charges.
Magnetostatic - field by charges in motion i.e. steady current 39

Electric charges - +ve & -ve
seperable.
Magnetic charges - Not possible to isolate magnetic charge i.e. north pole & south pole.

* When magnetic north pole exist, it is accompanied with south pole & net magnetic charge is always zero.

Source of magnetic field is steady current
* Electric field can exert a force on static charge as well as moving charge.
* Magnetic field exerts force only on moving charge & not on static charge.
* This force is given by -

$F = q(\bar{E} + \vec{v} \times \vec{B})$ - Lorentz force.

Current is scalar
(1) current is source of mag field
(2) It is amount of charge crossing boundary

② Ampears law.

$$\oint B \cdot dl = \int_S J \cdot ds$$

amp/m × m   amp/m² & m²

Diff. form

$$\nabla \times \vec{B} = \vec{J}$$

<u>Word</u>

Magnetic field ~~swirls~~ circulates around electric current

③ Faradays law.

$$\oint \vec{E} \cdot dl = -\frac{\partial}{\partial t}\int_S \vec{B} \cdot \vec{ds}.$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Electric field circulates around changing magnetic flux

④

$$\oint_S \vec{B} \cdot \vec{ds} = 0$$

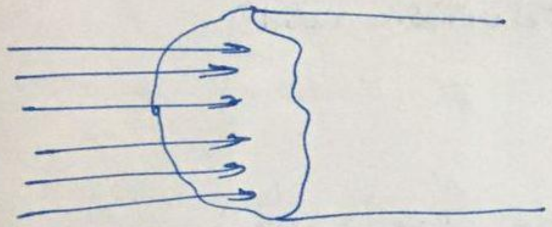
volume integral of magnetic charges.

$$\nabla \cdot B = 0.$$

Non existance of magnetic monopoles.

of a surface of volume per unit time
= Rate of change of charge in volume.

(3) Equation of continuity – In steady state there is no accumulation of charges in volume.



Depletion of charge S Increase of charges.

$$I = \oint \bar{J} \cdot d\bar{s}$$

J → current density.

$$\frac{dQ}{dt} = \frac{d}{dt}\int_V \rho \, dv$$

$$= \underbrace{\int_V \frac{d\rho}{dt} dv}_{\text{Increase of charges at right side}} = \underbrace{-\oint J \cdot ds}_{\text{Decrease of charges at left side.}}$$

Using divergence theorem on right side.

$$\int_V \frac{d\rho}{dt} dv = -\int_V (\nabla \cdot J) \, dv$$

The equation is valid for any arbitrary volume, thus integrands can be equated!

$$\therefore \frac{d\rho}{dt} + \nabla \cdot J = 0 \rightarrow \text{Eq}^n \text{ of continuity.}$$

It tells about conservation of charge i.e. if

Maxwells equations.

① $\oint E \cdot ds = \frac{q}{\epsilon_0}$ Gauss law in electrostatic

Elec. charge is source of electric field

② $\oint B \cdot ds = 0$ Gauss law in magnetostatic.

Incoming flux is equal to outgoing magnetic flux thus $\oint B \cdot ds = 0$.

monopole in magnetism not possible

Magnetic lines of force are continuous.

(Electric field lines are not continuous.

Faradays law of EMI

③ $e = -\frac{d\phi_B}{dt}$ Faradays law of electromagnetism

$\phi_B = B \cdot A$

$\boxed{\frac{dV}{dr} = E}$

$e = -\frac{d\phi_B}{dt} \Rightarrow E \cdot dl = A \frac{dB}{dt}$

emf.

Changing ~~of~~ magnetic field induce electric field.

$dV = E \cdot dr$.

Pot. diff.

$e = \int E \cdot dl$.

④ Ampear – Maxwell law.

change of electric field creates magnetic field.

$\oint \bar{B} \cdot dl = \mu_0 (i_c + i_d)$

$\oint \bar{B} \cdot d\bar{l} = \mu_0 i_c + \mu_0 \frac{d}{dt} \phi_E$

$\oint B \cdot dl = \mu_0 i_c + \mu_0 \frac{d}{dt} (E \cdot A)$

dE/dt

changing electric field creates changing magnetic field, but this changing magnetic field creates electric field.

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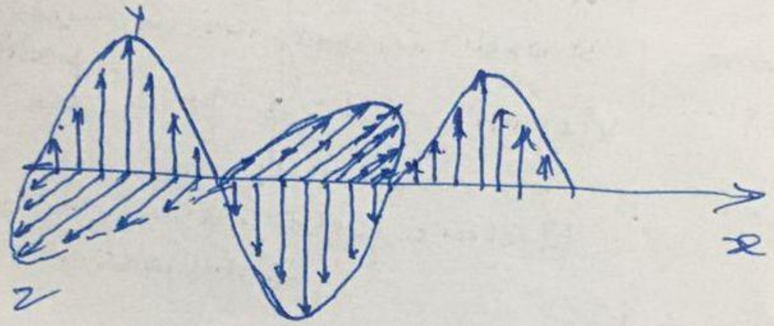

Maxwell's eqn

Integral form	diff. form	Words.
① $\oint_S E\,ds = \int_V \rho\, dv$	$\nabla \cdot E = \rho$	Electric charges spawn (come out) Electric flux. +ve charge spawn -ve charges sink ϕ